A Continuous Model for Two-Lane Traffic Flow

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PRIMES Conference
May 16, 2015

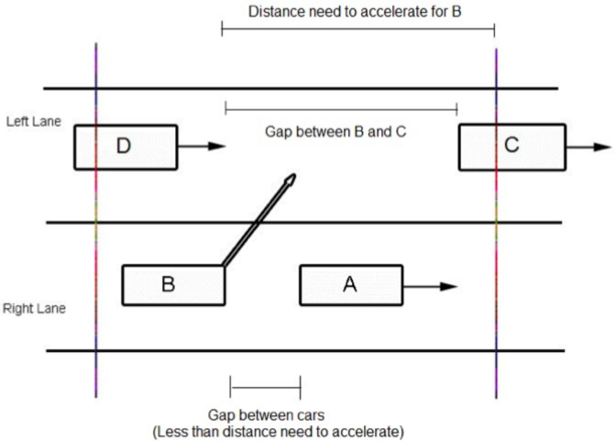
Two Ways of Approaching Traffic Flow

- Microscopic (discrete): study interactions between individual cars
  - Pro: Gives a more accurate representation of the flow
  - Con: Very computationally intensive, even for computers
- Macroscopic (continuous): properties of individual cars treated as insignificant to the overall flow
  - Pro: Can be represented using differential equations and general variables
  - Con: May not give accurate representation, especially in "extreme" situations

Current Microscopic Models

- **Cellular automata model**: Road is modeled by lattice, cars travel from one lattice to another
- **Car-following model**: Vehicles modify speed according to vehicles in front
- **Multi-class model**: Several classes of car types and drivers have different preferences
- For our model, we drew on ideas from the cellular automata and car-following models, but assumed all cars were functionally identical

The Microscopic View of Overtaking



For car B to change lanes, it must consider the locations and velocities of cars A, C, and D.

The Microscopic View of Overtaking

Gipps' Model:

Theorem

$v_n(t+r) =$
$\min(v_n(t) + 2.5a_n r(1 - v_n(t)/V_n)\sqrt{(0.025 + v_n(t)/V_n)}, b_n r +$
$\sqrt{(b_n r)^2 - b_n(2(x_{n-1}(t) - s_{n-1} - x_n(t)) - v_n(t)r - v_{n-1}(t)^2/b)})$

- Gipps' Model correctly accounts for the circumstances necessary for an individual car overtaking
- Accurate in theory but variables can be difficult to compute and track
- Used in this project to run a computer simulation
  - Needed reasonable estimations of some variables

The Macroscopic Model

Variables:

- Distance x, time t
- Velocity: $v(x, t) = \frac{\Delta x}{\Delta t}$
- Density over some interval: $\rho(x, t) = \frac{\text{Number of cars}}{\Delta x}$
  - Actual interval size is insignificant, but should not be too small or too large
- Flux: $J(x, t) = \rho(x, t)v(x, t)$
  - Number of cars passing through a certain point per unit time

New Model for Two-Lane Macroscopic Flow

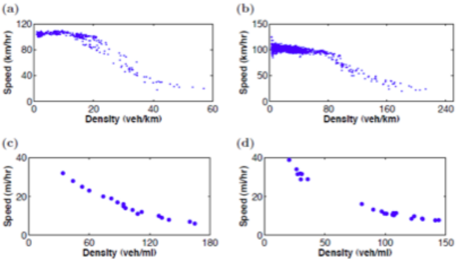
- Cars originating in lane 1 can move to lane 2 to overtake, but lane 2 cars cannot switch lanes
  - Modification of "Keep-Right-Except-to-Pass" model
- Lane 1 cars currently on lane 1 have velocity v_1, density ρ_1; those currently on lane 2 have velocity v_2, density ρ_2
- Lane 2 cars have velocity u and density β
- Density can be normalized such that $\rho_1 + \rho_2 = 1$, like a "probability"

Example: Constant density distribution

- Assume all cars have length L, distance d from each other in a two-lane highway
- Since density is constant, we could also assume velocity is constant and thus no overtaking
- Then, density is $\frac{1}{L+d}$ for every interval
- Total flux $J = J_1 + J_2 = \rho_1 v_1 + \rho_2 v_2 + \beta u$ is constant

New Model for Two-Lane Macroscopic Flow

- Vector fields for velocity and density
- Using Greenshields model
  - Velocity is linear to density for all cars: $v = v_m(1 - \frac{\rho}{\rho_m})$
  - Not considering individual drivers' preferences — v_m, ρ_m constants



The speed-density relationship for data taken from four roads in different countries.

Lighthill-Whitman-Richards Model

For single-lane flow, the following approximate relation holds:

Theorem

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial J(x,t)}{\partial x} = 0$$

- Proved using Fundamental Theorem of Calculus and conservation of vehicles

Intermediate Results

Theorem

The LWR model holds for multiple-lane flow as well; that is, for two lanes, $\frac{\partial}{\partial t}(\rho_1 + \rho_2) = -\frac{\partial}{\partial x}(\rho_1 v_1 + \rho_2 v_2)$

Definition

The *inviscid Burgers' equation* describes fluid flow with zero viscosity: $\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = 0$.

- Linear velocity model actually generated an additional $\frac{\partial^2 v}{\partial x^2}$ term

Potential Applications

- Operating an unmanned vehicle
- Developing ways to improve traffic systems and minimize jams
- Extending to model the movement of a fluid along a fixed pathway

Future Research

- Comparing our theoretical model to a computer simulation and real data
    - Take safety into account (e.g. unexpected braking)
- Removing the simplifying restriction on cars in Lane 2
- Improving the velocity model to make it more realistic

Acknowledgements

I would like to thank:

- Prof. Gabriele La Nave for helping me through the research process and taking the time to answer my questions
- the PRIMES program for this unique opportunity
- Dr. Tanya Khovanova for offering advice and guidance
- My parents for their continuous support in my endeavors